Exercise 2

Convert each of the following IVPs in 1–8 to an equivalent Volterra integral equation:

$$y' + 4xy = e^{-2x^2}, \ y(0) = 0$$

Solution

Let

$$y'(x) = u(x). \tag{1}$$

Integrate both sides from 0 to x.

$$\int_0^x y'(t) \, dt = \int_0^x u(t) \, dt$$

$$y(x) - y(0) = \int_0^x u(t) \, dt$$

Substitute $y(0) = 0$.

$$y(x) = \int_0^x u(t) \, dt \tag{2}$$

Substitute equations (1) and (2) into the original ODE.

$$y' + 4xy = e^{-2x^2} \quad \rightarrow \quad u(x) + 4x \left[\int_0^x u(t) \, dt \right] = e^{-2x^2}$$

Therefore, the equivalent Volterra integral equation is

$$u(x) = e^{-2x^2} - 4 \int_0^x xu(t) \, dt.$$